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Collatz conjecture

From Wikipedia, the free encyclopedia

The **Collatz conjecture** is a conjecture in mathematics that concerns a sequence defined as follows: start with any positive integer *n*. Then each term is obtained from the previous term as follows: if the previous term is even, the next term is one half the previous term. If the previous term is odd, the next term is 3 times the previous term plus 1. The conjecture is that no matter what value of *n*, the sequence will always reach 1.

Unsolved problem in mathematics:

Does the Collatz sequence eventually reach 1 for all positive integer initial values?

(more unsolved problems in mathematics)

The conjecture is named after Lothar Collatz, who introduced the idea in 1937, two years after receiving his doctorate.[1] It is also known as the **3*n* + 1 conjecture**, the **Ulam conjecture** (after Stanisław Ulam), **Kakutani's problem** (after Shizuo Kakutani), the **Thwaites conjecture** (after Sir Bryan Thwaites), **Hasse's algorithm** (after Helmut Hasse), or the **Syracuse problem**;[2][4] the sequence of numbers involved is referred to as the **hailstone sequence** or **hailstone numbers** (because the values are usually subject to multiple descents and ascents like hailstones in a cloud),[5][6] or as **wondrous numbers**.[7]

Paul Erdős said about the Collatz conjecture: "Mathematics may not be ready for such problems."[8] He also offered $500 for its solution.[9] Jeffrey Lagarias in 2010 claimed that based only on known information about this problem, "this is an extraordinarily difficult problem, completely out of reach of present day mathematics."[10]

Reminder: Racket functions

```
(define (int-average a b)
  (quotient (+ a b) 2))
```

(define (*function-name* *parameter*$_1$ … *parameter*$_n$)
    body)

Boolean functions

Write two functions:

odd? Takes a number and returns `true` if it is odd; `false` otherwise.

even? Takes a number and returns `true` if it is even; `false` otherwise.

You can assume that the functions take integers; you don't have to check.

Racket's modulo function may be useful.

Firstname Lastname **T. 10 / 9**

(Your response)

Boolean functions

Write two functions:

odd? Takes a number and returns true if it is odd; false otherwise.

even? Takes a number and returns true if it is even; false otherwise.

You can assume that the functions take integers; you don't have to check.

Racket's modulo function may be useful.

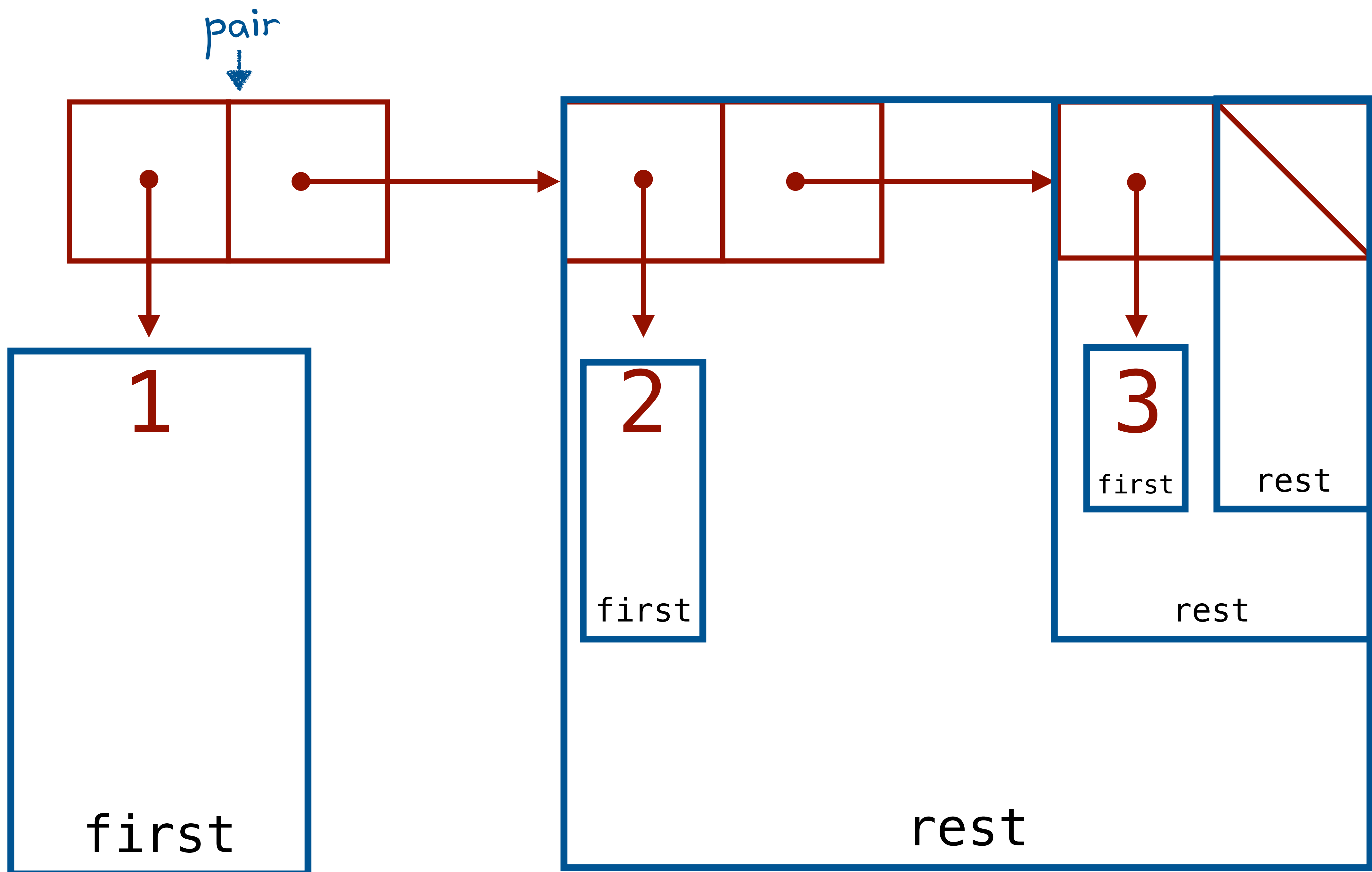
Firstname Lastname **T. 10 / 9**

```
(define (odd? n)
  (= (modulo n 2) 1))

(define (even? n)
  (not (odd? n)))
```

Note: odd? and even? are built-in Racket functions.

Reminder: Racket lists

```
(cons 1    (cons 2    (cons 3 '())))
```



Watch out!

Don't do these things (and if you accidentally do, know how to recognize them)

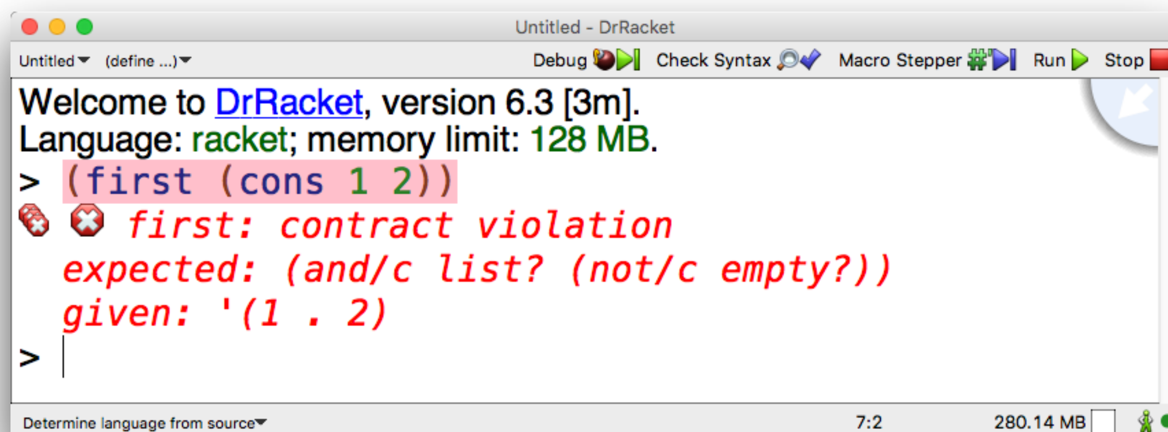
```
> (cons 1 2)
'(1 . 2)
```

←not a list!

This expression builds a *pair.*

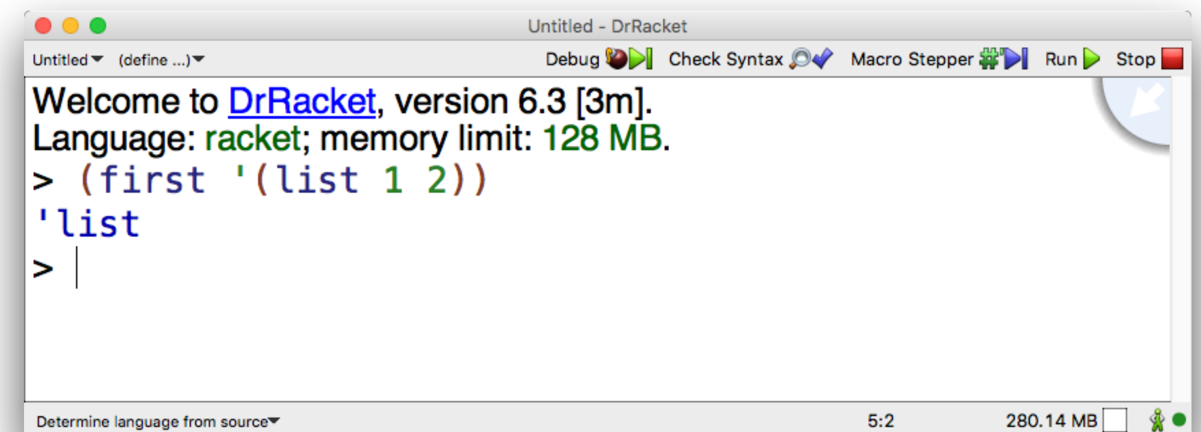
A pair is **not** a list.

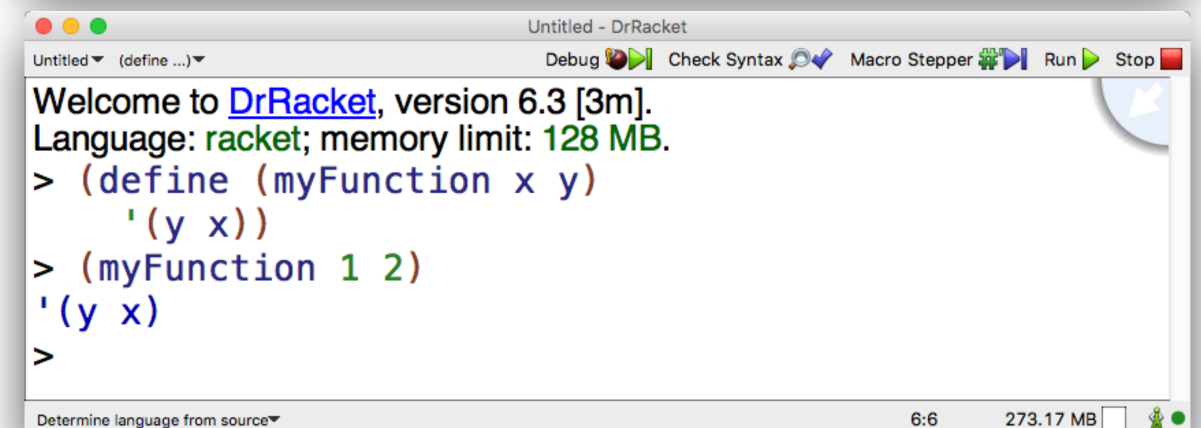
You can't call `first` on it.
You can't call `rest` on it.




```
> '(list 1 2)
'(list 1 2)
```

This expression builds a list whose first element is `'list`!





For more info, see: docs.racket-lang.org/reference/pairs.html

Recursive functions over lists

```
;; len
;;    inputs: a list, L
;;    outputs: the number of elements in the list
(define (len L)



                                     )
```

Recursive functions over lists

```
;; len
;;   inputs: a list, L
;;   outputs: the number of elements in the list
(define (len L)



                                    )


; tests
(check-equal? (len '()) 0)
(check-equal? (len '(1 2 3)) 3)
(check-equal? (len '((1 2 3))) 1)
```

Recursive functions over lists

```
;; len
;;   inputs: a list, L
;;   outputs: the number of elements in the list
(define (len L)

                                    )

; tests
(check-equal? (len '()) 0)
(check-equal? (len '(1 2 3)) 3)
(check-equal? (len '((1 2 3))) 1)
```

base case

recursive step

Recursive functions over lists

```
;; len
;;   inputs: a list, L
;;   outputs: the number of elements in the list
(define (len L)
  (if (empty? L)                          base case

                                  ))      recursive step


; tests
(check-equal? (len '()) 0)
(check-equal? (len '(1 2 3)) 3)
(check-equal? (len '((1 2 3))) 1)
```

Recursive functions over lists

```
;; len
;;   inputs: a list, L
;;   outputs: the number of elements in the list
(define (len L)
  (if (empty? L)
      0
                                ))
```

base case

recursive step

```
; tests
(check-equal? (len '()) 0)
(check-equal? (len '(1 2 3)) 3)
(check-equal? (len '((1 2 3))) 1)
```

Recursive functions over lists

```
;; len
;;   inputs: a list, L
;;   outputs: the number of elements in the list
(define (len L)
  (if (empty? L)
      0
      (+ 1 (len (rest L)))))      recursive step

; tests
(check-equal? (len '()) 0)
(check-equal? (len '(1 2 3)) 3)
(check-equal? (len '((1 2 3))) 1)
```

base case

recursive step

Let’s practice: `sum`

```
;; sum
;;   inputs: a list of integers, L
;;   outputs: the sum of the integers in L
(define (sum L)





; tests
(check-equal? (sum '()) 0)
(check-equal? (sum '(1)) 1)
(check-equal? (sum '(1 2 1)) 4)
(check-equal? (sum '(1 2 3 4)) 10)
```

Let's practice: `sum`

```
;; sum
;;   inputs: a list of integers, L
;;   outputs: the sum of the integers in L
(define (sum L)
  (if (empty? L)
      0
      (+ (first L) (sum (rest L)))))


; tests
(check-equal? (sum '()) 0)
(check-equal? (sum '(1)) 1)
(check-equal? (sum '(1 2 1)) 4)
(check-equal? (sum '(1 2 3 4)) 10)
```

Let's practice: `remove`

```
;; remove
;;   inputs: an integer i
;;           a list of integers, L
;;   outputs: a new list with 1st occurrence
;;            of i removed
(define (remove e L)
```

hint: two base cases

```
; tests
(check-equal? (remove 1 '()) '())
(check-equal? (remove 1 '(1)) '())
(check-equal? (remove 1 '(1 1)) '(1))
(check-equal? (remove 4 '(1 2 3)) '(1 2 3))
```

Let's practice: `remove`

```
;; remove
;;   inputs: an integer i
;;           a list of integers, L
;;   outputs: a new list with 1st occurrence
;;            of i removed
(define (remove e L)
  (cond [(empty? L) '()]
        [(= (first L) e) (rest L)]
        [else (cons (first L) (remove e (rest L)))]))


; tests
(check-equal? (remove 1 '()) '())
(check-equal? (remove 1 '(1)) '())
(check-equal? (remove 1 '(1 1)) '(1))
(check-equal? (remove 4 '(1 2 3)) '(1 2 3))
```

theplatypusblog.files.wordpress.com/2013/05/close-up-of-a-female-duck-billed-platypus-with-two-eggs-ornithorhynchus-anatinus1.jpg

Implement these functions

Using list operations (`cons`, `empty`, `empty?`, `first`, and `rest`) and recursion

`plus-one`: given a list of integers L produces a new list L' where all the corresponding elements are one larger

```
> (plus-one '(1 2 3 4))
'(2 3 4 5)
```

`double`: given a list of integers L produces a new list L' where all the corresponding elements of L are doubled

```
> (double '(1 2 3 4))
'(2 4 6 8)
```

`odds`: given a list of integers L produces a new list L' that contains only the odd integers from L

```
> (odds '(1 2 3 4))
'(1 3)
```

`bigs`: given a list of integers L produces a new list L' that contains only the integers from L that are larger than 30.

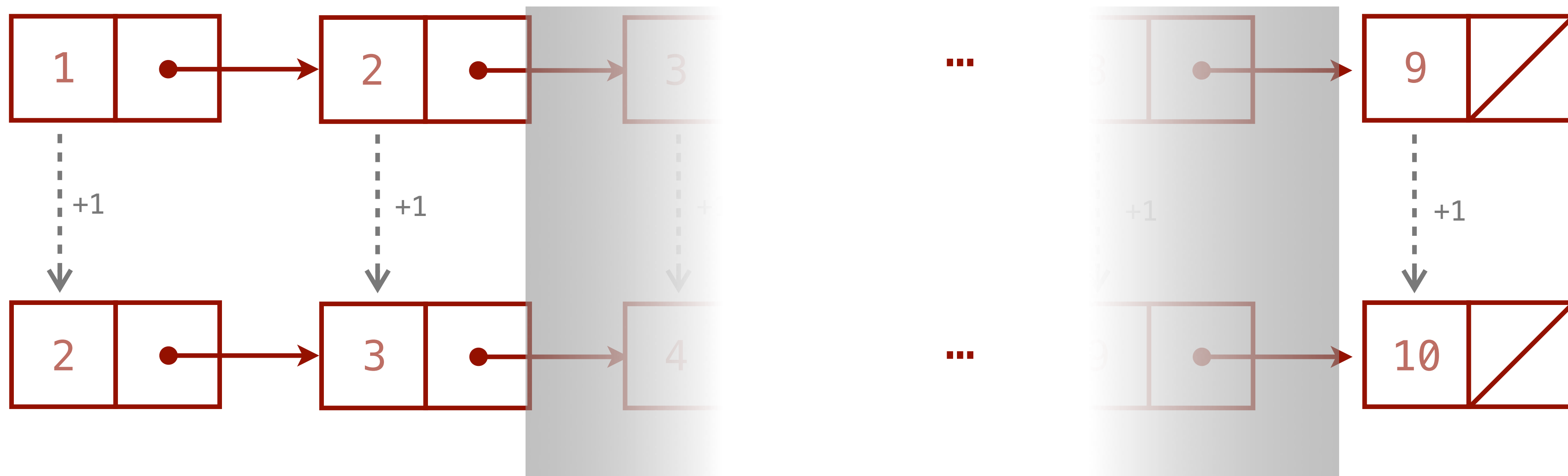
```
> (bigs '(100 1 42 35))
'(100 42 35)
```

`product`: given a list of integers L computes the product of all the integers.

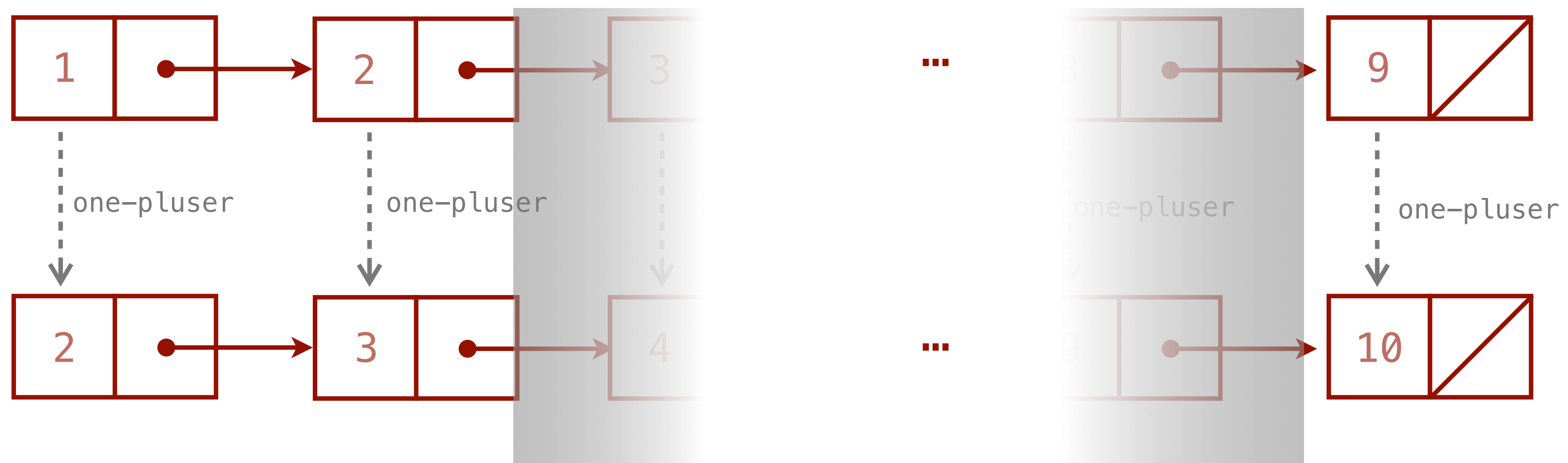
```
> (product '(1 2 3 4))
24
```

You can assume that the inputs to these functions are lists of integers.

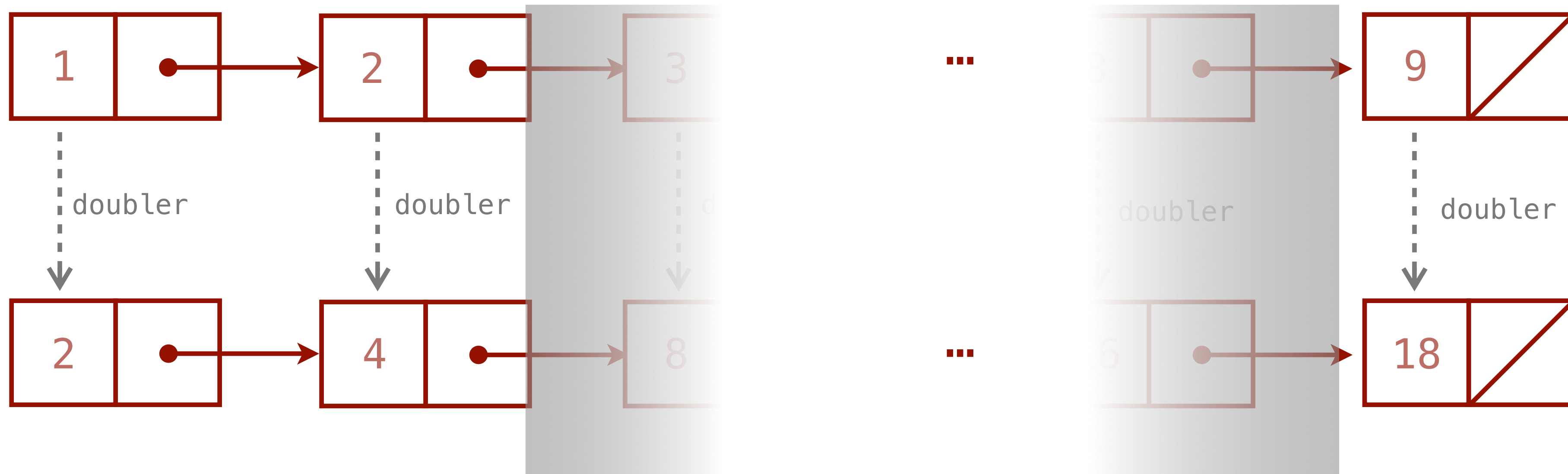
```
(define (plus-one L)
  (if (empty? L)
      '()
      (cons (+ 1 (first L)) (plus-one (rest L)))))
```



```
(define (one-pluser n) (+ n 1))

(define (plus-one L)
  (if (empty? L)
      '()
      (cons (one-pluser (first L)) (plus-one (rest L)))))
```



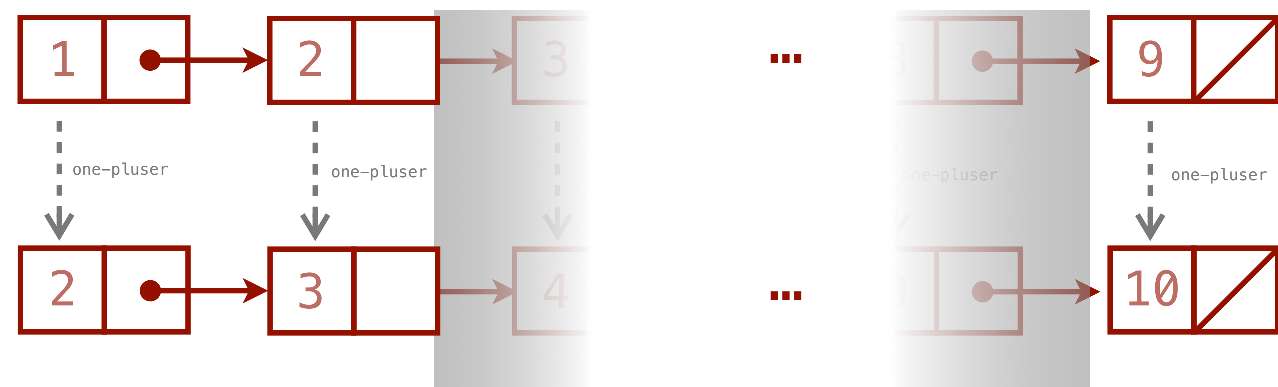
```
(define (doubler    n) (* n 2))

(define (double   L)
  (if (empty? L)
      '()
      (cons (doubler    (first L)) (double   (rest L)))))
```



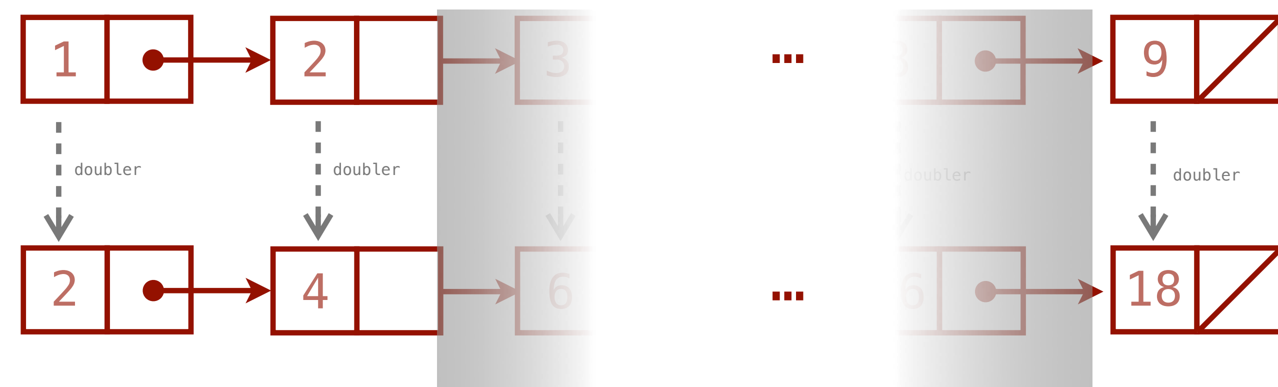
A common pattern

Apply a transformer function to each element of a list

```
(define (one-pluser n) (+ n 1))

(define (plus-one L)
  (if (empty? L)
      '()
      (cons (one-pluser (first L)) (plus-one (rest L)))))
```




```
(define (doubler n) (* n 2))

(define (double L)
  (if (empty? L)
      '()
      (cons (doubler (first L)) (double (rest L)))))
```



Higher-order functions

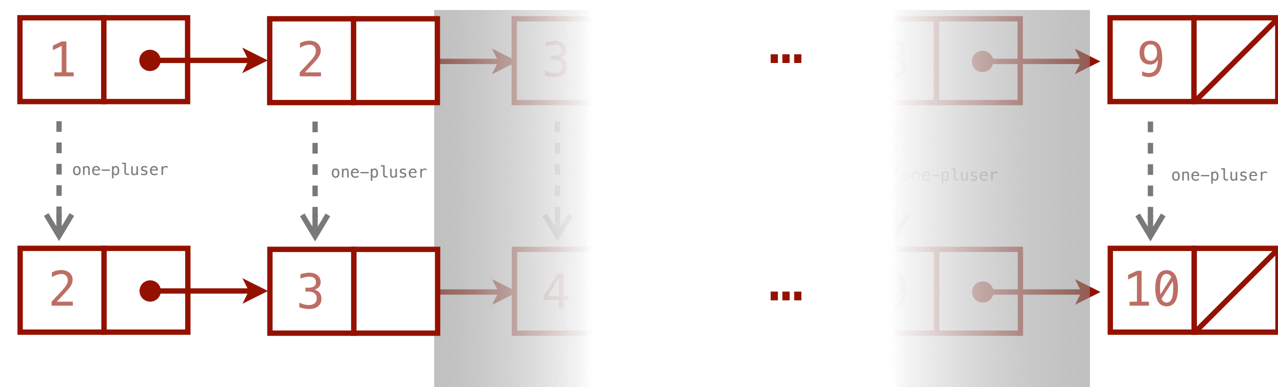
(functions as values)

A **higher-order function** is a function that takes at least one function as an argument *or* that returns a function as its result (or both).

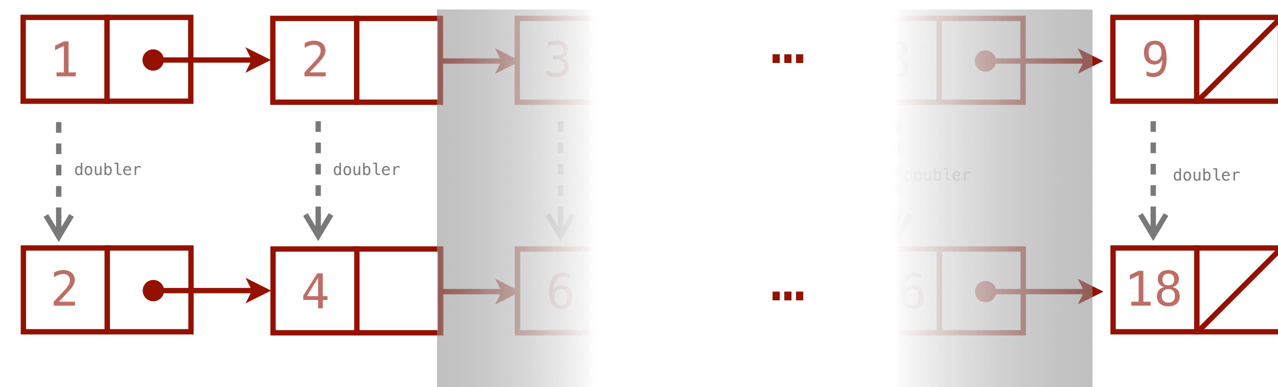
A common pattern

Apply a transformer function to each element of a list

```
(define (one-pluser n) (+ n 1))

(define (plus-one L)
  (if (empty? L)
      '()
      (cons (one-pluser (first L)) (plus-one (rest L)))))
```




```
(define (doubler n) (* n 2))

(define (double L)
  (if (empty? L)
      '()
      (cons (doubler (first L)) (double (rest L)))))
```

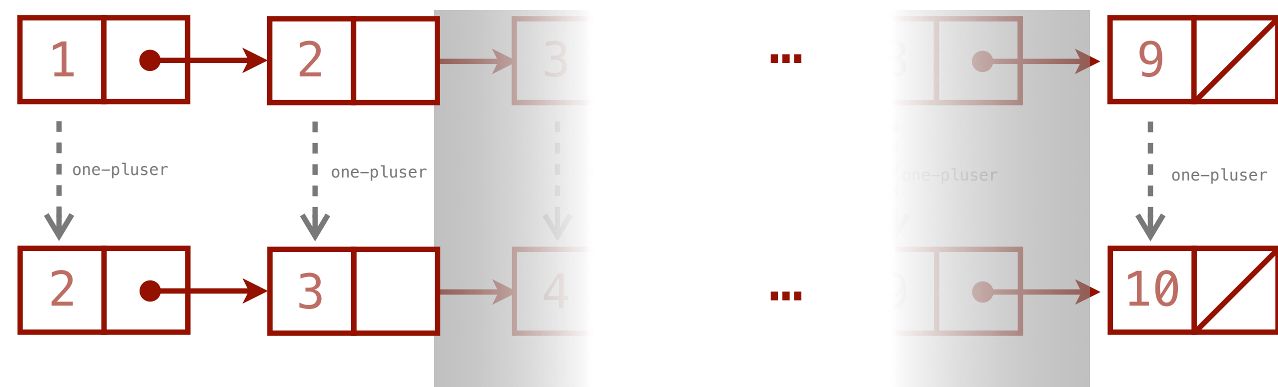


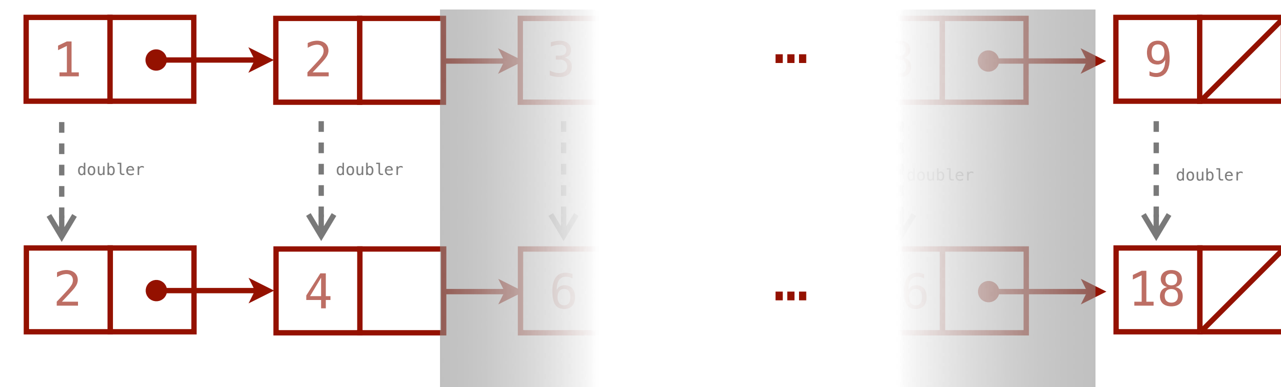
map

Apply a transformer function to each element of a list

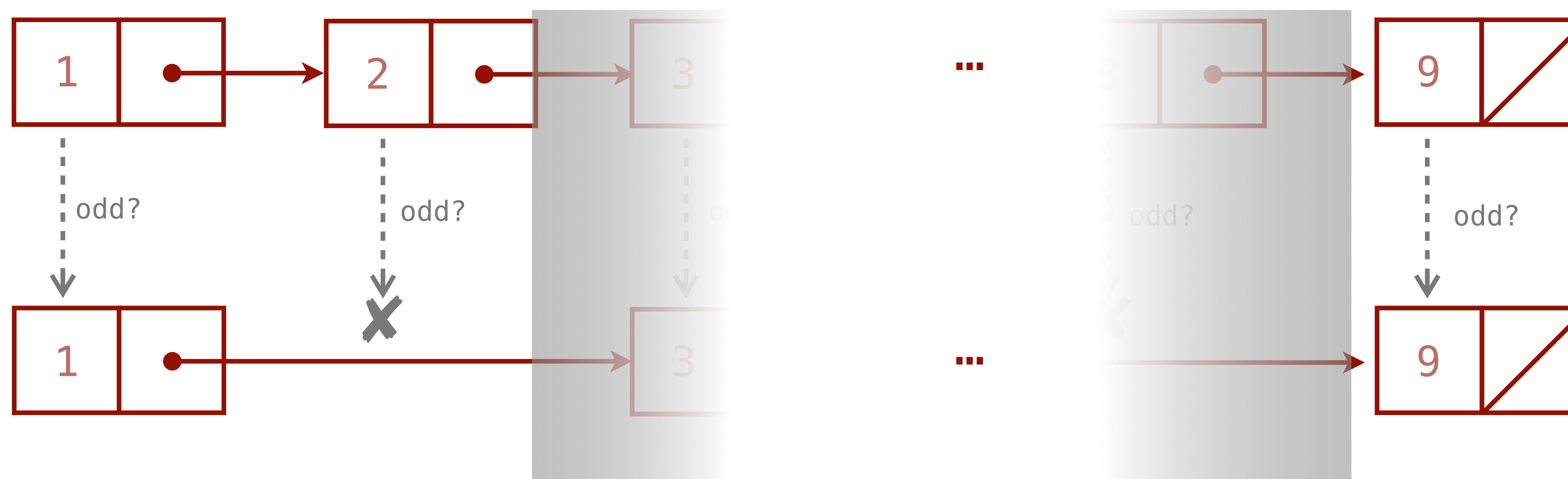
```
(define (one-pluser n) (+ n 1))

(define (plus-one L)
   (map one-pluser L))
```

```
(define (doubler n) (* n 2))

(define (double L)
   (map doubler L))
```

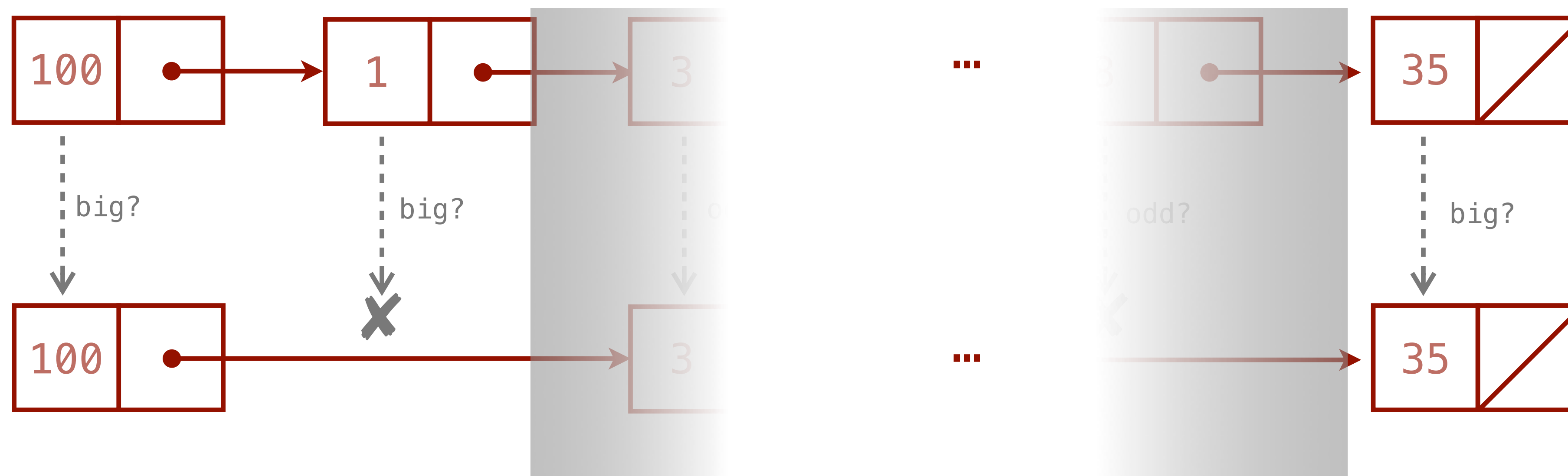




```
(define (odd?  n) (= (modulo n 2) 1))

(define (odds  L)
  (cond [(empty? L) '()]
        [(odd?  (first L)) (cons (first L) (odds  (rest L)))]
        [else (odds  (rest L))]))
```



```
(define (big?  n) (> n 30))

(define (bigs  L)
  (cond [(empty? L) '()]
        [(big?  (first L)) (cons (first L) (big  (rest L)))]
        [else (big  (rest L))]))
```

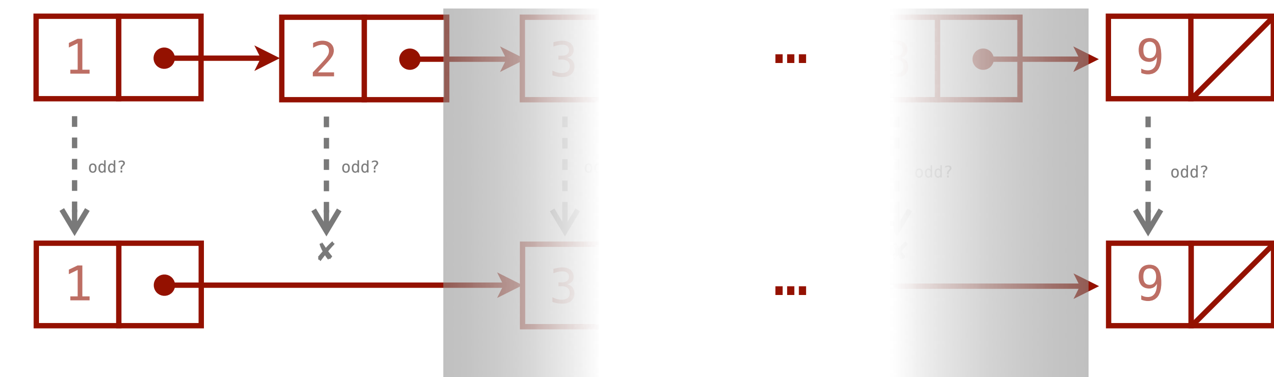


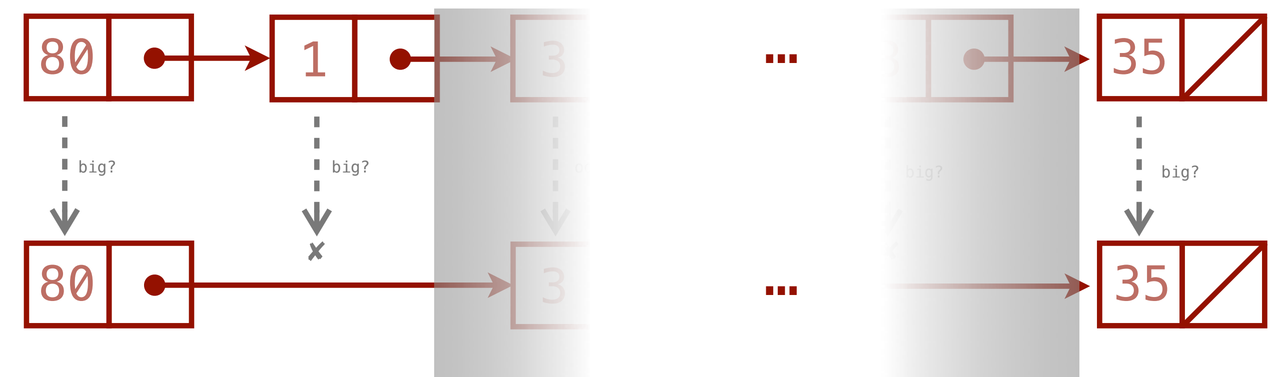
A common pattern

Use a predicate function to cull a list

```
(define (odd? n) (= (modulo n 2) 1))

(define (odds L)
  (cond [(empty? L) '()]
        [(odd? (first L)) (cons (first L) (odds (rest L)))]
        [else (odds (rest L))]))
```

```
(define (big? n) (> n 30))

(define (bigs L)
  (cond [(empty? L) '()]
        [(big? (first L)) (cons (first L) (bigs (rest L)))]
        [else (bigs (rest L))]))
```

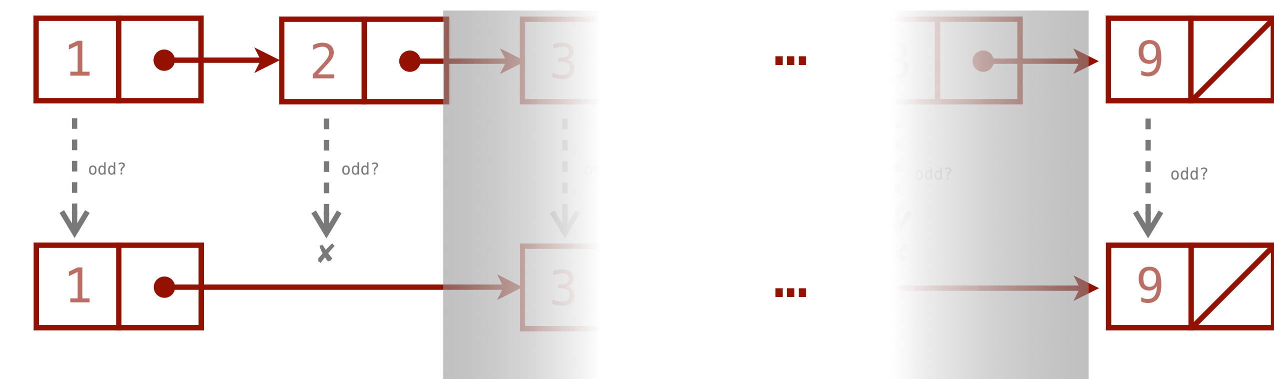




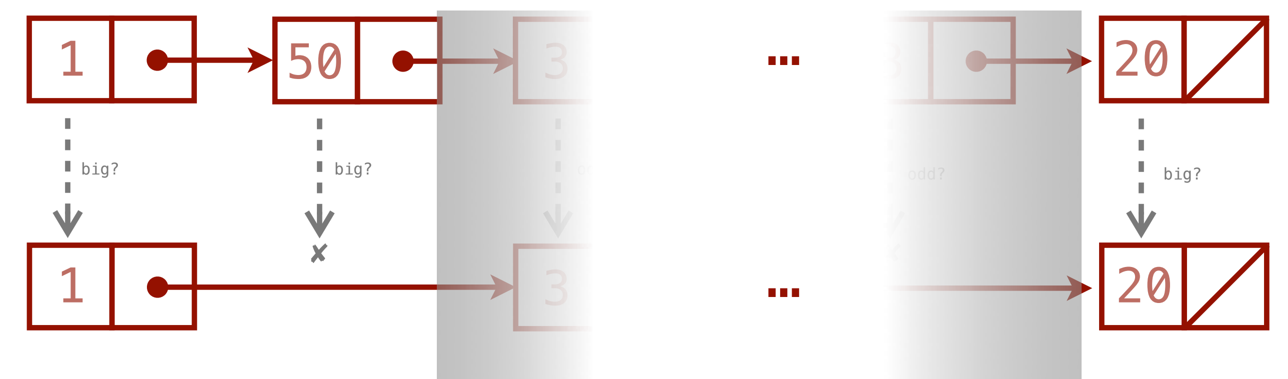
filter

Use a predicate function to cull a list

```
(define (odd? n) (= (modulo n 2) 1))

(define (odds L)
  (filter odd? L))
```




```
(define (big? n) (> n 30))

(define (big L)
  (filter big? L))
```



```
(define (sum      L)
  (if (empty? L)
      0
      (+ (first L) (sum      (rest L)))))
```

```
(define (product L)
  (if (empty? L)
      1
      (* (first L) (product (rest L)))))
```

A common pattern

Reduce a list to a value by accumulating the list's elements

```
(define (sum L)
  (if (empty? L)
      0
      (+ (first L) (sum (rest L)))))
```

```
(define (product L)
  (if (empty? L)
      1
      (* (first L) (product (rest L)))))
```

watch out for order of operations!

$$(\texttt{foldl f seed L}) \equiv (\texttt{f } v_n \texttt{ (... (f } v_0 \texttt{ seed) ...))}$$

foldl

Reduce a list to a value by accumulating the list's elements (start at the beginning of list and move towards end of the list)

```
(define (sum L)
  (foldl + 0 L))
```

```
(define (product L)
  (foldl * 1 L))
```

foldr

Reduce a list to a value by accumulating the list's elements (start at the end of list and move towards the beginning of the list)

watch out for order of operations!

```
(foldr f seed L) ≡ (f v0 (… (f vn seed) … ))
```

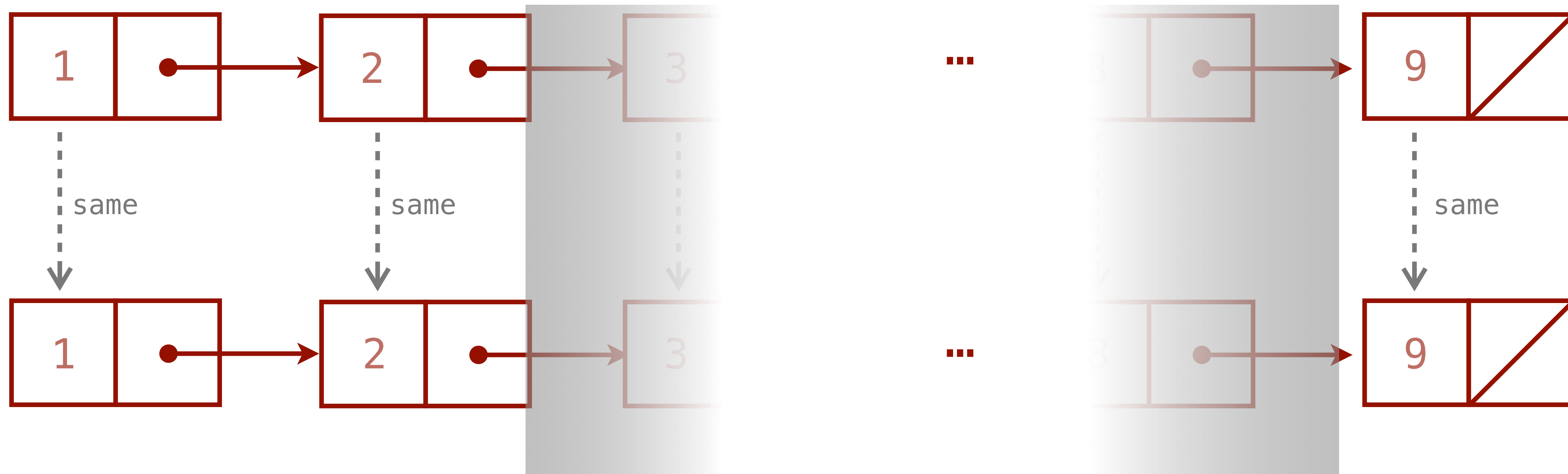
```
(define (sum L)
  (foldr + 0 L))
```

```
(define (product L)
   (foldr * 1 L))
```

A common pattern

Define one function to be used as an argument to another function

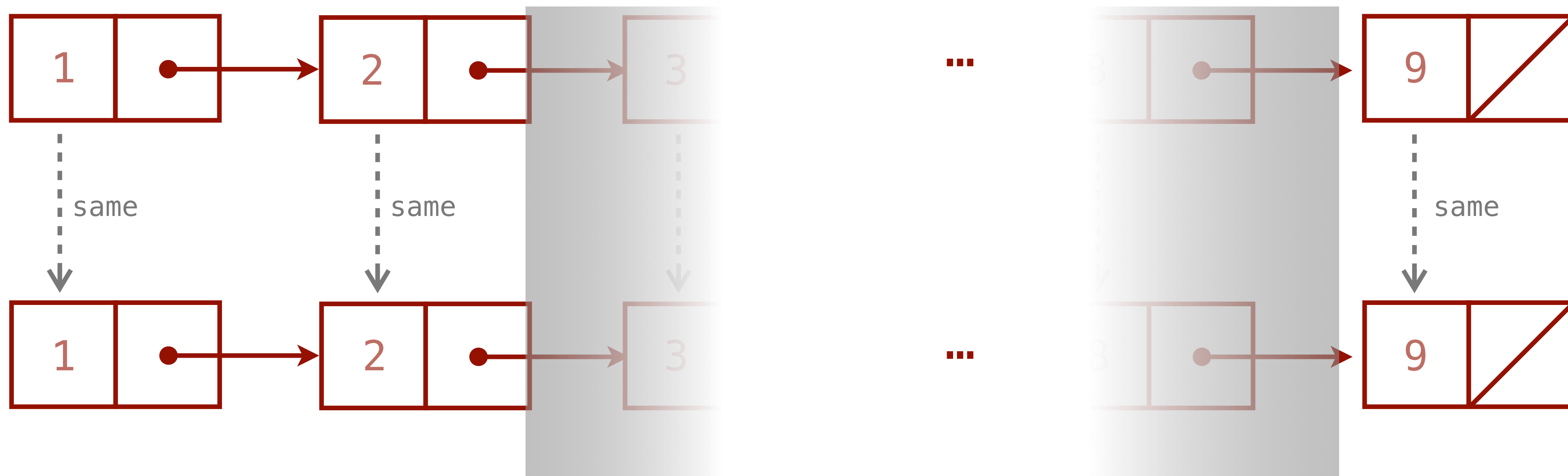
```
(define (one-pluser n) (+ n 1))

(define (plus-one L)
   (map one-pluser L))
```



Anonymous functions (λ)

Define one function to be used as an argument to another function

```
(define (plus-one L)
   (map (lambda (x) (+ x 1)) L))
```



Anonymous functions (λ)

```
(define (one-pluser x) (+ x 1))
```

≡

```
(define one-pluser (lambda (x) (+ x 1)))
```

Implement these functions

Using `map`, `filter`, `foldl`, `foldr`, and `lambda`

`times3`: multiplies each number in a list L by 3.

```
> (times3 '(1 2 3 4))
'(3 6 9 12)
```

`9multiples`: filters a list L to remove any numbers that aren't divisible by 9.

```
> (9multiples '(8 9 10))
'(9)
```

`sum-of-times10`: multiplies each element of a list L by 10, then sums the results

```
> (sum-of-times10 '(1 2 3))
60
```

`count-ones`: counts the number of times the value 1 appears in a list L

write one version using a combination of `filter` and `foldr`
write another version using only `foldr`

```
> (count-ones '(1 2 1))
2
```

click here for solutions

Bonus

`copy`, **`reverse`**

with `map`? with `foldl` or `foldr`?

`map`, **`filter`**, **`foldr`**, **`foldl`**

You can assume that the inputs to these functions are lists of integers.

A higher-order puzzle

Define a function called `divisible-by` that can be used as follows:

```
> (filter (divisible-by 2) (range 10))
'(0 2 4 6 8)
> (filter (divisible-by 3) (range 10))
'(0 3 6 9)
> (filter (divisible-by 4) (range 10))
'(0 4 8)
> (filter (divisible-by 5) (range 10))
'(0 5)
```

It may help to know the following functions:

```
> (range 10)
'(0 1 2 3 4 5 6 7 8 9)

> (modulo 10 2)
0
```

```
(define (divisible-by k)
  (lambda (n) (= 0 (modulo n k))))
```

The divisible-by function returns another function!

Assignment

Ciphers as a way to study lists and HOFs

Can you decrypt this message?

```
xjwtmhjxmjnfyzyjzftunjjksdumqswrytjqndtuff
irunssfhiwtyyxttgflgujjxynnsxqsntnlcnmtsdn
jiswjwysjsyfytfjfhjjlkxmsnkiryjjskztjitxfn
jxtytskxtszulftkvwiuhkbhymskhjnfnnrjmfyirt
zsryjzwjyhxzraxjhsxjcnthxudysmhnxutrnnqzfy
jjhzwsujhjfsfitfxjxtstcvhqxkqhxyxyknd
```

Strings can be interpreted as lists of characters

How to write a single character in Racket

```
> (string->list "Hello world!")
'(#\H #\e #\l #\l #\o #\space #\w #\o #\r #\l #\d #\!)

> (list->string
   '(#\H #\e #\l #\l #\o #\space #\w #\o #\r #\l #\d #\!))
"Hello world!"
```