“Programmers waste enormous amounts of time thinking about, or worrying about, the speed of noncritical parts of their programs, and these attempts at efficiency actually have a strong negative impact when debugging and maintenance are considered. We should forget about small efficiencies, say about 97% of the time: premature optimization is the root of all evil. Yet we should not pass up our opportunities in that critical 3%.”

—Don Knuth
Write the tabulation template for fib

<table>
<thead>
<tr>
<th>Name</th>
<th>(your response)</th>
<th>Th. 11/8</th>
</tr>
</thead>
</table>


make-change, infinite coins
Given a value and a set of coins, what is the minimum number of coins required to sum to the value, assuming we have an infinite number of each coin?
The longest-common substring of \texttt{s1} and \texttt{s2} is the longest string that is a non-consecutive substring of both \texttt{s1} and \texttt{s2}.

\begin{align*}
lcs('x', 'y') &= 0 & lcs('car', 'cat') &= 2 \\
lcs('x', '') &= 0 & lcs('human', 'chimpanzee') &= 4 \\
lcs('', 'x') &= 0
\end{align*}
Theoretical tools: code → math

How many times does the platypus quack?

```c
for (int i=0; i<N; i++) {
    for (int j=0; j<N; j++) {
        platypus.quack();
    }
}
```
Theoretical tools: code → math

How many times does the platypus quack?

```python
for j in range(N):
    platypus.quack()
```
Theoretical tools: code $\rightarrow$ math

summations

\[ \sum_{i=1}^{5} i = 1 + 2 + 3 + 4 + 5 \]
Theoretical tools: code → math

summations

\[
\sum_{j=0}^{N-1} 1
\]
Theoretical tools: code $\rightarrow$ math

Summations

\[ \sum_{j=0}^{N-1} 1 \]

(implicit increment)

Upper-bound (inclusive)

Index

<table>
<thead>
<tr>
<th>j</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>N-2</td>
<td>1</td>
</tr>
<tr>
<td>N-1</td>
<td>1</td>
</tr>
</tbody>
</table>

\( N \in \mathcal{O}(N) \)
Theoretical tools: code → math

summations

\[ \sum_{i=1}^{N} 1 \]

<table>
<thead>
<tr>
<th>i</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>N-1</td>
<td>1</td>
</tr>
<tr>
<td>N</td>
<td>1</td>
</tr>
</tbody>
</table>

\( N \in O(N) \)
Theoretical tools: code $\rightarrow$ math

summations

$$\sum_{i=1}^{N} N$$

<table>
<thead>
<tr>
<th>$i$</th>
<th>cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$N$</td>
</tr>
<tr>
<td>2</td>
<td>$N$</td>
</tr>
<tr>
<td>3</td>
<td>$N$</td>
</tr>
<tr>
<td>4</td>
<td>$N$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>N-1</td>
<td>$N$</td>
</tr>
<tr>
<td>N</td>
<td>$N$</td>
</tr>
</tbody>
</table>

$N^2 \in O(N^2)$
Theoretical tools: code $\rightarrow$ math

summations

\[
\sum_{i=1}^{N} i
\]

$\in O(N^2)$
Theoretical tools: code → math

How many times does the platypus quack?

```python
for j in range(N):
    platypus.quack()
```

Math:

\[
\sum_{j=0}^{N-1} 1
\]

Sum 1, j=0 to N-1

Wolfram Alpha

closed form

\( N \)

Asymptotic notation

\( O(N) \)
Theoretical tools: code → math

How many times does the platypus quack?

```python
for i in range(N):
    for j in range(N):
        platypus.quack()
```
Theoretical tools: code $\rightarrow$ math

How many times does the platypus quack?

**code**

```python
for i in range(N):
    for j in range(N):
        platypus.quack()
```

**math**

$$\sum_{i=0}^{N-1} \sum_{j=0}^{N-1} 1$$

sum (sum 1,j=0 to N-1),i=0 to N-1

**Wolfram Alpha**

$N^2$

**closed form**

**asymptotic notation**

$O(N^2)$
Theoretical tools: code $\rightarrow$ math

How many times does the platypus quack?

```python
for i in range(N):
    for j in range(i, N):
        platypus.quack()
```
Theoretical tools: code $\rightarrow$ math

How many times does the platypus quack?

```python
for i in range(N):
    for j in range(i, N):
        platypus.quack()
```

```
\sum_{i=0}^{N-1} \sum_{j=i}^{N-1} 1
```

Wolfram Alpha

```
\text{sum (sum 1, } j=\text{i to N-1}, \text{i=0 to N-1)}
```

Closed form

```
\frac{N(N+1)}{2}
```

Asymptotic notation

```
O(N^2)
```